ALEVEL MATHEMATICS SEMINAR 2018

MECHANICS

1.(a) A particle moving in a straight line with a uniform acceleration $a\,ms^{-2}$ passes certain point with a velocity $u\,ms^{-1}$. 3 s later another particle, moving in the same straight line with a constant acceleration $4/_3\,a\,ms^{-2}$, passes the same point with a velocity of $1/_3\,u\,ms^{-1}$. The first particle is overtaken by the second particle when their velocities are $8.1ms^{-1}$ and $9.3ms^{-1}$ respectively. Find the;

(i) value of u and a

(ii) Distance travelled from the point

(b) A particle of weight 20N rests on a rough plane inclined at 30^0 to the horizontal, the coefficient of friction between the particle and the plane being 0.25. Find the horizontal force required;

(i) To just prevent the particle from slipping down

(ii) To make the particle just begin to slide up.

2.(a) Show that the position of C.O.G of a uniform solid hemisphere of radius r is $\frac{3r}{8}$ from the straight edge.

(b) A child's toy consist of a solid uniform hemisphere of radius r and a solid right circular cone of base radius r and height h. The base radius of the solids are glued together. If the density of the hemisphere is k times that of the cone,

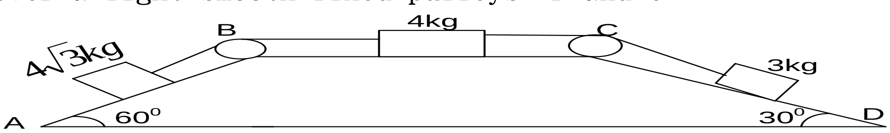
(i) show that the distance from the vertex of the cone to the centre of gravity of the toy is

$$\frac{kr(3r+8h)+3h^2}{4(2kr+h)}$$

If the toy is suspended from a point on the rim of the common base and rests in equilibrium with the axis of the cone inclined at an angle of θ to the downward vertical. Show that $tan\theta = \frac{4r(2kr+h)}{h^2-3kr^2}$

3.(a) Two smooth spheres A and B of equal radii and masses 180g and 100g respectively are travelling along the same horizontal line. The initial speeds of A and B are 2m/s and 6m/s respectively. The spheres collide and after collision both spheres reverse their directions and B moves with a speed of 3m/s. Find the speed of A after collision and loss in kinetic energy of the system.

(b) The diagram below shows a 4kg mass on a horizontal rough plane with coefficient of friction 0.25. The $4\sqrt{3}$ kg mass rests on a smooth plane inclined at angles of 60^0 to the horizontal while the 3kg rests on a rough plane inclined at an angle 30^0 to the horizontal and coefficient of friction $\frac{1}{\sqrt{3}}$. The masses are connected to each other by a light inextensible strings passing over a light smooth fixed pulleys B and C



(a)Find the

(i) Acceleration of the system

(ii) Tension in the strings

(b)Find the work done against friction if each particle travels a distance of 0.5m

4.(a) A vertical tower stands on a level ground. A stone is thrown from the top of the tower and has an initial velocity of 24.5m at an angle of $tan^{-1}\left(\frac{4}{3}\right)$ above the horizontal. The stone strikes the ground at a point 73.5m from the foot of the tower. Find the;

(i) time taken for the stone to reach the ground

(ii) height of the tower

(b) A particle is projected from a point O on the level ground, with initial speed un/s to pass through a point which is a horizontal distance **a** from O and a distance **b** vertically above the level O

(i) show that there are two possible angles of projection

(ii) If these angles are α and β, prove that $tan(\alpha+\beta) = -\frac{a}{b}$

take $g = 10ms^{-2}$

5.(a) ABCD is a square of side 3m. Forces of magnitude 1N, 2N, 3N, sN, and tN act along the line AB, BC, CD, DA, and AC respectively, in each case the direction of the force being given by the order of the letters. Taking AB as horizontal and BC as vertical, find the values of s and t so that the resultant of the forces is a couple.

(b) ABCDEF is a regular hexagon of side 2a. Forces of magnitude 3N, 4N, 2N, 1N, uN and vN act along the line AB, BC, CD, DE, EF and FA respectively, in each case the direction of the force being given by the order of the letters. Find the resultant of the value of u and v if the resultant of the six forces acts along CE

NUMERICAL METHODS AND ERRORS

6. (a) Show that the equation $tanx + x = 0$ has a real root between $x = 2$ and $x = 3$

(b) Using linear interpolation to find the first approximation for the root (a) above, correct your answer to 2dp

(c) Use Newton Raphson method to find the value of the root of the equation $tanx + x = 0$, use the first approximation in (b) above correct your answer to 2dp

7.(a) Show that the iterative formula base on Newton Raphson's method for finding the natural logarithm of a number N is given by

$$x_{n+1} = \frac{e^{x_n}(x_n - 1) + N}{e^{x_n}}, \qquad n = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots$$

(b) Draw a flow chart that;

(i)Reads N and the initial approximation x_0 of the root

(ii) Computes and prints the natural logarithm after four iterations and gives the natural logarithm to three decimal places

(c) Taking, $N = 10, x_0 = 2$, perform a dry run for the flow chart, give your root correct to three decimal places

8. (a) Use the trapezium rule with five sub intervals to estimate the area enclosed by the curve $y = \mathrm{x}^2 e^x$, the x-axis, $x = 0$, and $x = 1$, Give your answer correct to 3dp.

(b) (i) find the exact value of $\int_0^1 \mathrm{x}^2 e^x\, dx$

(ii) find the relative error in your estimation

Suggest how the error may be reduced

9. Given that $A = \quad xysin\theta$

(a) Deduce that the maximum possible relative error in A is given by

$$\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right| + |\Delta\theta| cot\theta$$

where $\Delta x, \Delta y\ and\ \Delta\theta$ are small numbers compared to x, y and θ respectively

(b) find the percentage error made in the area, if x and y are measured with errors of ± 0.5 and angle with an error of $\pm 0.5^0$ given that $x = 2.5cm$, $y = 3.4cm$ and $\theta = 30°$

10. The numbers M and N are approximated with possible errors of e_1 and e_2 respectively.

(a) Show that the maximum absolute error in the quotient $\frac{M}{N}$ is given by

$$\left|\frac{\mathrm{M}}{N}\right| \left\{ \left|\frac{e_1}{M}\right| + \left|\frac{e_2}{N}\right| \right\}$$

(b) A car covers a distance of $6.43km$ with an average speed of $37.2kmh^{-1}$, where the quantities are rounded off to the given decimal point, find the range within which the time the bicycle takes in travel lies.

STATISTICS

11. The frequency distribution table shows the weight of 100 children measured to the nearest kg

weight	<15	<20	<25	<30	<40	<50
Number of children	5	14	26	44	84	100

(a)Calculate the;

(i) mean and

(ii) standard deviation

(iii)mode

(b)Draw a cumulative frequency curve and use it to estimate

(i) Median

(ii) Semi interquartile range

(iii) Number of children with weight above 37kg

12.(*b*) The table below shows the price relatives for the year 2012, 2015 and 2017 together with their weights for a certain family

Item	Weight	Price (shs)		
		2012	2015	2017
food	35	100	98	125
water	11	100	102	121
housing	8	100	105	112
Electricity	6	100	100	108
Clothing	22	100	115	118

(i) Using 2012 as the base year, calculate the value of the cost of living index for 2015 and 2017

(ii) Given that a certain item costs 56000/= in 2012, determine its likely cost in 2015 and 2017 respectively **An(103.93, 120,)**

(*b*) The table below shows the prices (in Ug Shs) of some food items in January, June and December together with the corresponding weights.

Item	Weight	Price (shs)		
		Jan	Jun	Dec
Matooke (1 bunch)	4	15,000	13,000	18,000
Meat (1kg)	1	6,500	6,000	7,150
Posho (1kg)	3	2,000	1,800	1,600
Beans (1kg)	2	2,200	2,000	2,800

Taking January as the base month, calculate the;

(i) Simple aggregate price index for June

(ii) Weighted aggregate price index for December

An((i) 88.72, (ii) 116.61)

13. Eight candidates seeking admission to a certain school sat for mathematics and physics exams. The scores were shown below.

Maths (x)	55	54	35	62	87	53	71	50
Physics (y)	57	60	47	65	83	56	74	63

(a)Plot the results on a scatter diagram. Comment on the relationship between the mathematics and physics

(b)Draw the line of best fit on your graph and use it to estimate y when $x = 70$ An(71)

(c)Calculate the rank correlation coefficient. Comment on the significance of the result at 5% significant level An($\rho = 0.833$)

14.(a) Box P contains 4 red and 3 green sweets and box Q contains 5 red and 6 green sweets. A box is randomly selected and 2 sweets are randomly picked from it, one at a time without replacement. If P is twice as likely to be picked as Q, find the probability that both sweets are

(i) of the same colours,

(ii) from P given that they are of same colours.

(iii) expected number of red sweets removed

(b) The continuous random variable X has a cumulative distribution function F(x) where

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \alpha x & 0 \leq x \leq 1 \\ \dfrac{x}{3} + \beta & 1 \leq x \leq 2 \\ 1, & x \geq 2 \end{cases}$$

Find the

(i) Value of α and β

(ii) Probability density function, f(x) and sketch it

(iii) Mean of X and variance of X

(iv) $P(X < 1.5 / X > 1)$

An (i) $\alpha = \frac{2}{3}$, $\beta = \frac{1}{3}$ $(iii)\ \mu = \frac{5}{6}, Var(X) = \frac{19}{36}$ $(iv) = 0.4998$

15.(a) The marks of an examination were normally distributed. 20% of the students scored below 40 marks while 10% of the students scored above 75 marks

(i) Find the mean mark and standard deviation of the students

(ii) If 25 students were chosen at random from those who sat for the examination, what is the probability that their average mark exceeds 60

(iii) If a sample of 8 students were chosen, find the probability that not more than 3 scored between 45 and 65 marks An((a) μ =53.87, σ =16.473, $(\boldsymbol{b})$ =0.0313, $(\boldsymbol{c}) = \mathbf{0.5419}$)

(iv) If a random sample of 60 students is taken from the population calculate the 95% confidence interval for μ based on the sample

(b) Events A and B are such that $P(A\ U\ B) = 0.8$, $P(A/_B) = 0.2$ and $P(A^1\ n\ B) = 0.4$. Find

(i) $P(A\ n\ B)$

(ii) $P(B)$

(iii) $P(A)$

(iv) $P\left(A/_{B^1}\right)$

An $\boldsymbol{(i) = 0.1\ \ (ii) = 0.5\ \ (iii) = 0.4\ \ (iv) = 0.6, (v) = 0.4}$

Solutions

1. (a) $v^2 = u^2 + 2as$

For 1st particle: $(8.1)^2 = u^2 + 2as$

$$65.61 - u^2 = 2as \dots\dots\dots\dots (i)$$

For 2nd particle: $(9.3)^2 = \left(u\,{}^{1}/_{3}\right)^2 + 2\text{x}\,{}^{4}/_{3}\,as$

$$86.49 - \frac{u^2}{9} = \frac{8}{3}as$$

$$\frac{778.41 - u^2}{9} = \frac{8}{3}as \dots\dots\dots\dots (ii)$$

$$(ii) \div (i)$$

$$\left(\frac{778.41 - u^2}{9}\right)\Big/(65.61 - u^2) = \frac{8}{3}as\Big/2as$$

$$\frac{778.41 - u^2}{9}\text{x}3 = (65.61 - u^2)x4$$

$$778.41 - u^2 = (262.44 - 4u^2)3$$

$$778.41 - u^2 = 787.32 - 12u^2$$

$$11u^2 = 8.91$$

$$u^2 = 0.81$$

$$u = 0.9ms^{-1}$$

Also $v = \text{u} + \text{at}$

For 1st particle: $8.1 = 0.9 + \text{aT}$

$$7.2 = \text{aT} \dots\dots\dots\dots \text{(iii)}$$

For 2nd particle: $9.3 = {}^{1}/_{3}\,x0.9 + {}^{4}/_{3}\,a\,(\text{T} - 3)$

$$9 = {}^{4}/_{3}\,\text{aT} - 4\text{a} \dots\dots\dots\dots \text{(iv)}$$

Put (iii) into (iv)

$$9 = {}^{4}/_{3}\,\text{x}7.2 - 4\text{a}$$

$$-0.6 = -4a$$

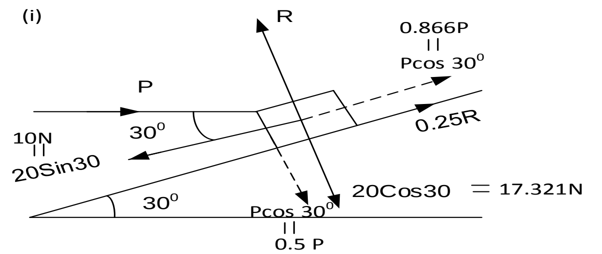
$$a = 0.15ms^{-2}$$

$$v^2 = u^2 + 2as$$

For 1st particle: $8.1^2 = 0.9^2 + 2x0.15s$

$$s = 216m$$

(b)



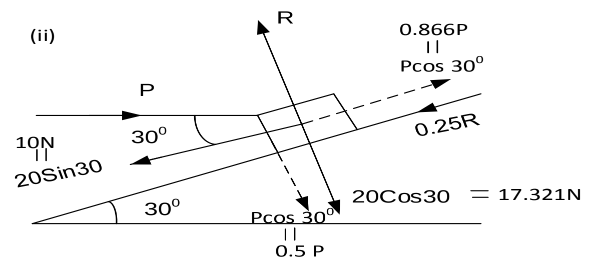
$$R = 0.5\text{P} + 17.321$$

$$10 = 0.866P + 0.25R$$

$$10 = 0.866P + 0.25(0.5\text{P} + 17.321)$$

$$10 - 4.33025 = 0.866P + 0.125P$$

$$P = 5.721N$$



$$R = 0.5\text{P} + 17.321$$

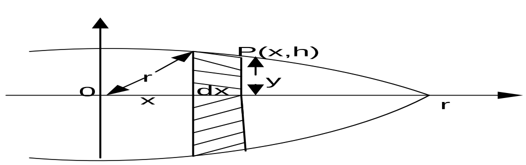
$$10 = 0.866P - 0.25R$$

$$10 = 0.866P - 0.25(0.5\text{P} + 17.321)$$

$$10 + 4.33025 = 0.866P + 0.125P$$

$$P = 19.3391N$$

2 (a)



Vol of a hemisphere=volof the element

$$W\frac{2}{3}\pi r^3\bar{x} = W\pi\int_0^r xy^2dx$$

But $x^2 + y^2 = r^2$

$$y^2 = r^2 - x^2$$

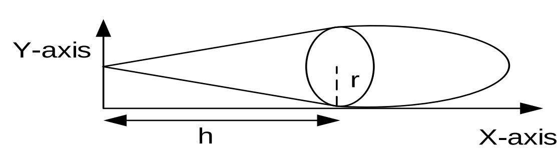
$$\frac{2}{3}\pi r^3\bar{x} = \pi\int_0^r x(r^2 - x^2)dx$$

$$\frac{2}{3}r^3\bar{x} = \left[r^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^r$$

$$\frac{2}{3}r^3\bar{x} = \frac{r^4}{2} - \frac{r^4}{4}$$

$$\bar{x} = \frac{3r}{8}$$

2 (b)(i)



Let w be weight per unit volume of a cone and wk weight per unit volume of a hemisphere

lamina	Weight	C.O.G from y axis
Cone	$\frac{1}{3}\pi r^2 hW$	$\frac{3}{4}h$
hemisphere	$\frac{2}{3}\pi r^3 Wk$	$h+\frac{3r}{8}$
Composite	$\frac{1}{3}\pi r^2(h+2rk)W$	$\bar{x}$

$$\frac{1}{3}\pi r^2(h+2rk)W\bar{x} = \frac{2}{3}\pi r^3\left(h+\frac{3r}{8}\right)kW + \frac{1}{3}\pi r^2 h\left(\frac{3h}{4}\right)W$$

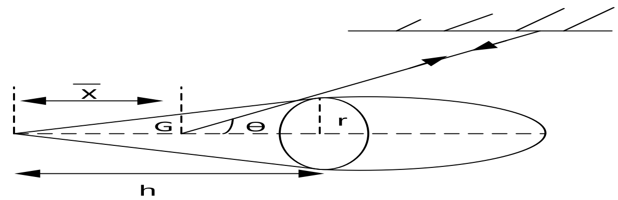
$$(h+2\mathrm{k}r)\bar{x} = 2rk\left(h+\frac{3r}{8}\right)+\frac{3h^2}{4}$$

$$\bar{x} = \frac{16\mathrm{rkh}+6\mathrm{k}r^2+6h^2}{8(h+2\mathrm{k}r)}$$

$$\bar{x} = \frac{8\mathrm{rkh}+3\mathrm{k}r^2+3h^2}{8(h+2\mathrm{k}r)}$$

$$\bar{x} = \frac{kr(3r+8h)+3h^2}{4(2kr+h)}$$

2 (b)(ii)



C.O.G from the base$= h - \frac{kr(3r+8h)+3h^2}{4(2kr+h)}$

$$= \frac{8krh+4h^2-3kr^2-8krh-3h^2}{4(2kr+h)}$$

$$= \frac{h^2-3kr^2}{4(2kr+h)}$$

$$tan\theta = \frac{r}{\left(\frac{h^2-3kr^2}{4(2kr+h)}\right)} = \frac{4r(2kr+h)}{h^2-3kr^2}$$

3.

(a) $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$$0.18x2 + 0.1x - 6 = 0.18x{-}v_A + 0.1x3$$

$$v_A = 3ms^{-1}$$

initial k.e$= \frac{1}{2}x0.18x2^2 + \frac{1}{2}x0.1x(-6)^2$

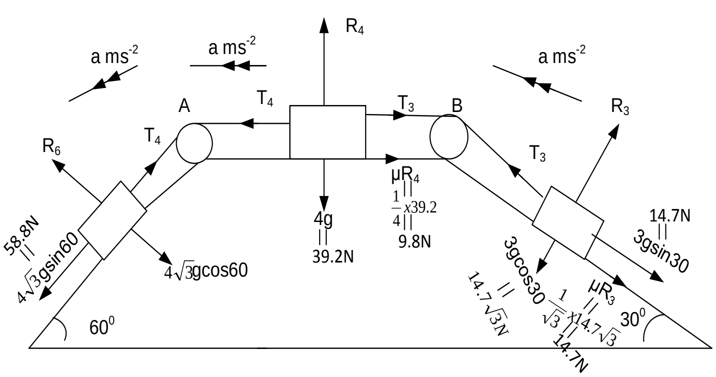
$$= 2.16J$$

final k.e$= \frac{1}{2}x0.18x(-3)^2 + \frac{1}{2}x0.1x(3)^2$

$$= 1.26J$$

loss in k.e$= 2.16 - 1.260.9J$

(b)



3kg mass: $T_3 - 14.7 - 14.7 = 3a$

$T_3 - 29.4 = 3a$.....**(i)**

4kg mass: $T_4 - T_3 - 9.8 = 4a$.......**(ii)**

$4\sqrt{3}$kg mass: $58.8 - T_4 = 4\sqrt{3}a$.........**(iii)**

(ii) + (i)

$T_4 - 39.2 = 7a$.........**(iv)**

(iii) + (iv)

$$58.8 - 39.2 = \left(7+4\sqrt{3}\right)a$$

$$a = 1.4072ms^{-2}$$

Tension:

$T_3 - 29.4 = 3a$

$T_3 = 3\text{x}1.4072 + 29.4 = 33.6217\text{N}$

$T_4 - 39.2 = 7a$

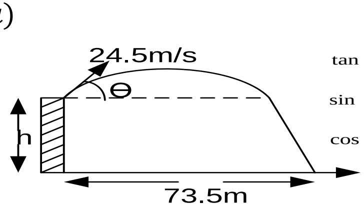
$T_4 = 7x1.4072 + 39.2 = 49.0504N$

Work done:

$$W = Fd$$

$$W = (14.7+9.8)x0.5 = 12.25J$$

4(a)



$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

Time of flight $t = 2.54s$

For horizontal motion:

$$x = ucos\theta xt$$

$$73.5 = 24.5\text{x}\frac{3}{5}\text{t}$$

$$t = 5s$$

For vertical motion: $y = usin\theta t - ½\, gt^2$

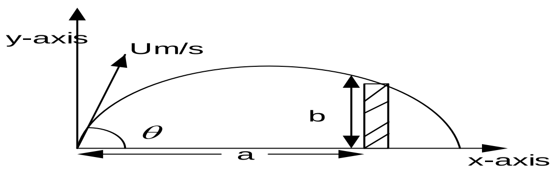
$$h = 24.5x\frac{4}{5}x5 - \frac{9.8(5)^2}{2}$$

$$= 98 - 122.5$$

$$h = -24.5m$$

24.5 m below the point of projection

4(*b*)



$$y = xtan\theta - \frac{g\,x^2(\,1 + tan^2\theta)}{2\,u^2}$$

$$b = atan\theta - \frac{10xa^2(\,1 + tan^2\theta)}{2xu^2}$$

$$b = atan\theta - \frac{5a^2}{u^2}(\,1 + tan^2\theta)$$

$$bu^2 = au^2tan\theta - 5a^2(\,1 + tan^2\theta)$$

$$5a^2tan^2\theta - au^2tan\theta + (bu^2 + 5a^2) = 0$$

since its a quadratic equation in $tan\theta$, its has two roots and two values of $\theta < 90$

$$tan\alpha + tan\beta = \frac{au^2}{5a^2}\ldots\ldots\ldots..(i)$$

$$tan\alpha x tan\beta = \frac{(bu^2 + 5a^2)}{5a^2}\ldots\ldots\ldots(ii)$$

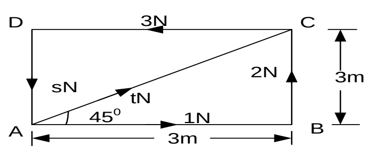
$$tan(\alpha + \beta) = \frac{tan\alpha + tan\beta}{1 - tan\alpha x tan\beta}\ldots\ldots(iii)$$

$$tan(\alpha + \beta) = \frac{\left(\frac{au^2}{5a^2}\right)}{1 - \left(\frac{bu^2 + 5a^2}{5a^2}\right)}$$

$$= \frac{\left(\frac{au^2}{5a^2}\right)}{\left(\frac{5a^2 - bu^2 + 5a^2}{5a^2}\right)} = \left(\frac{au^2}{5a^2}\right)x\frac{5a^2}{-bu^2}$$

$$tan(\alpha + \beta) = -\frac{a}{b}$$

5(a)



$$R = \binom{1}{0} + \binom{0}{2} + \binom{-3}{0} + \binom{0}{-s} + \binom{tcos45}{tsin45}$$

$$R = \binom{0}{0}$$

$(\rightarrow)\quad tcos45 = 2$

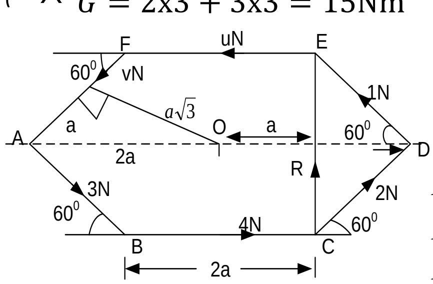
$$t = \frac{2}{cos45} = 2\sqrt{2}N$$

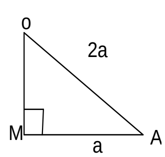
$(\uparrow)\quad 2 - s + tsin45 = 0$

$$s = 2 + 2\sqrt{2}sin45 = 4N$$

It must also be shown that $G \neq 0$

A $G = 2x3 + 3x3 = 15\text{Nm}$





$$MO = \sqrt{(2a)^2 - a^2}$$

$$MO = \sqrt{4a^2 - a^2}$$

$$MO = a\sqrt{3}$$

Since the resultant is vertical, its horizontal component is zero

$(\rightarrow)\quad 4 - u + (2 - 1 - v + 3)cos60 = 0$

$$(4 - v)x0.5 = u - 4$$

$$2u + v = 12\ldots\ldots\ldots..(i)$$

$(\uparrow)\quad (2 + 1 - v - 3)sin60 = R$

$$-v\frac{\sqrt{3}}{2} = R\ldots\ldots\ldots\ldots.(ii)$$

O $Ra = (3 + 4 + 2 + 1 + u + v)a\sqrt{3}$

$$R = (10 + u + v)\sqrt{3}\cdots\cdots\cdots\cdots(iii)$$

Equating (ii) and (iii)

$$-v\frac{\sqrt{3}}{2} = (10 + u + v)\sqrt{3}$$

$$2u + 3v = -20\ldots\ldots\ldots(iv)$$

(iv)-(i)

$$2v = -32$$

$$v = -16$$

Also $2u + v = 12$

$$2u - 16 = 12$$

$$2u = 28$$

$$u = 14$$

6(a)

$$f(x) = tanx + x$$
$$f(3) = tan(3) + 3 = 2.857$$
$$f(2) = tan(2) + 2 = -0.185$$

Since there is a sign change, then the root lies between 1 and 2

6(b)

x	1	x_0	2
f(x)	2.857	0	-0.185

$$\frac{x_0 - 1}{0 - 2.857} = \frac{2 - 1}{-0.185 - 2.857}$$
$$x_0 = 1.94$$

6(c) $f(x) = tanx + x$

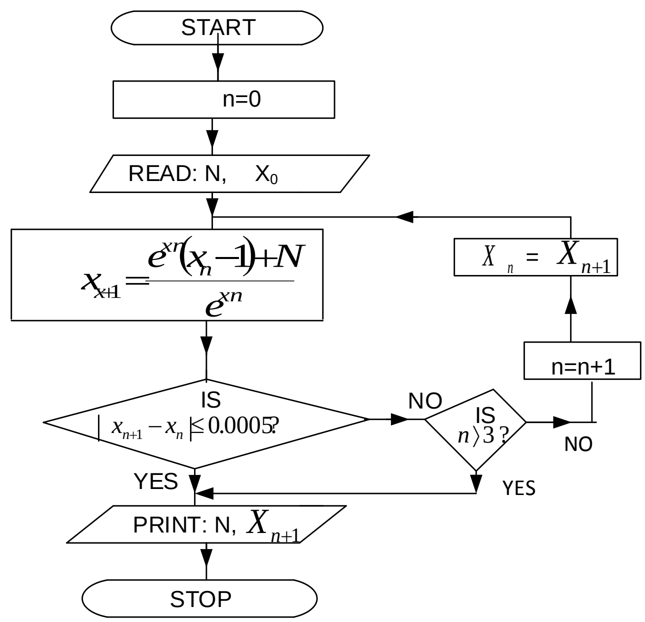
$$f^1(x) = sec^2x + 1$$
$$x_{n+1} = x_n - \left(\frac{tanx_n + x_n}{sec^2x_n + 1}\right)$$
$$x_{n+1} = \frac{x_n sec^2x_n + x_n - tanx_n - x_n}{sec^2x_n + 1}$$
$$x_{n+1} = \frac{x_n\left(\frac{1}{cos^2x_n}\right) - \left(\frac{sinx_n}{cosx_n}\right)}{\left(\frac{1}{cos^2x_n} + 1\right)}$$
$$x_{n+1} = \frac{\left(\frac{x_n - cosx_n sinx_n}{cos^2x_n}\right)}{\left(\frac{1 + cos^2x_n}{cos^2x_n}\right)}$$
$$= \frac{x_n - cosx_n sinx_n}{1 + cos^2x_n}$$
$$x_{n+1} = \frac{x_n - 0.5sin2x_n}{1 + cos^2x_n} \quad n = 0,1,2,3 \dots\dots$$
$$x_0 = 1.94$$
$$x_1 = \frac{1.94 - 0.5sin2\text{x}1.94}{1 + cos^21.94} = 2.014$$
$$x_2 = \frac{2.014 - 0.5sin2\text{x}2.014}{1 + cos^22.014} = 2.028$$
$$x_3 = \frac{2.028 - 0.5sin2\text{x}2.028}{1 + cos^22.028} = 2.029$$

Root= 2.03

7(a)

$$x = \text{In N} \quad \therefore \mathrm{e}^x = N$$
$$\mathrm{e}^x - N = 0$$
$$f(x) = \mathrm{e}^x - N, \qquad f^1(x) = \mathrm{e}^x$$
$$x_{n+1} = x_n - \left(\frac{\mathrm{e}^{x_n} - N}{\mathrm{e}^{x_n}}\right) = \frac{x_n\mathrm{e}^{x_n} - \mathrm{e}^{x_n} + N}{\mathrm{e}^{x_n}}$$
$$x_{n+1} = \frac{\mathrm{e}^{x_n}(x_n - 1) + N}{\mathrm{e}^{x_n}} \qquad n = 0,1,2,3 \dots\dots$$

7(b)



7(c)

n	X_n	X_{n+1}	$\lvert X_{n+1} - X_n \rvert$
0	2.0	2.3533	0.3533
1	2.3533	2.3039	0.0494
2	2.3039	2.3026	0.0013
3	2.3026	2.3026	0.0000

Root =2.303

8. (a)

$$h = \frac{1-0}{5} = 0.2$$

x	x^2e^x	
0	0	
0.2		0.0489
0.4		0.2387
0.6		0.6560
0.8		1.4243
1.0	2.7183	
Sum	2.7183	2.3679

$$\int_0^1 x^2e^x\,dx \approx \frac{1}{2}x0.2[2.7183 + 2(2.3679)]$$

$$\approx 0.74541 \approx 0.745$$

$$8(b)(i)\int_0^1 x^2e^x\,dx = [x^2e^x - 2xe^x + 2e^x]_0^1$$

$$= (1^2e^1 - 2x1e^1 + 2e^1) - (0^2e^0 - 2x0e^0 + 2e^0)$$

$$= 0.718$$

$(b)(ii)$ $error = |exact\ value - approx\ value|$

$$error = |0.718 - 0.745| = 0.027$$

$$R.E = \frac{error}{exact\ value} = \frac{0.027}{0.718} = 0.038$$

(b)(iii) Error is reduced by increasing the number of sub intervals

$9(a)$

$$A = \quad xysin\theta$$

$$e_A = \Delta x f^1(x) + \Delta y f^1(y) + \Delta\theta f^1(\theta)$$

$$e_A = (\Delta x)\text{ysin}\theta + (\Delta y)\text{xsin}\theta + (\Delta\theta)\text{xycos}\theta$$

$$|e_A| = |(\Delta x)\text{ysin}\theta + (\Delta y)\text{xsin}\theta + (\Delta\theta)\text{xycos}\theta|$$

$$\leq |\Delta x|\text{ysin}\theta + |\Delta y|\text{xsin}\theta + |\Delta\theta|\text{xycos}\theta$$

$$e_{max} = |\Delta x|\text{ysin}\theta + |\Delta y|\text{xsin}\theta + |\Delta\theta|\text{xycos}\theta$$

$$R.E = \frac{|\Delta x|\text{ysin}\theta + |\Delta y|\text{xsin}\theta + |\Delta\theta|\text{xycos}\theta}{xysin\theta}$$

$$R.E = \frac{|\Delta x|\text{ysin}\theta}{xysin\theta} + \frac{|\Delta y|\text{xsin}\theta}{xysin\theta} + \frac{|\Delta\theta|\text{xycos}\theta}{xysin\theta}$$

$$= \left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right| + |\Delta\theta|cot\theta$$

$$9\text{(b)}\ \text{R.E} = \left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right| + |\Delta\theta|cot\theta$$

$$= \left|\frac{0.05}{2.5}\right| + \left|\frac{0.05}{3.4}\right| + \left|\frac{0.5}{180}\pi\right| cot30°$$

$$R.E = 0.0492$$

$$\%\,R.E = 0.0492x100\% = 4.92\%$$

$10.(a)$

$$e_{M/N} = \frac{(M+e_1)}{(N+e_2)} - \left(\frac{M}{N}\right)$$

$$e_{M/N} = \frac{MN + Ne_1 - Me_2 - MN}{N^2 + Ne_2}$$

$$e_{M/N} = \frac{Ne_1 - Me_2}{N^2\left(1 + \frac{e_2}{N}\right)}$$

Since e_1 and e_2 are very small, then

$$\frac{e_2}{N} \approx 0$$

$$\left|e_{M/N}\right| = \left|\frac{Ne_1 - Me_2}{N^2}\right|$$

$$\left|e_{M/N}\right| \leq \frac{|Ne_1| + |Me_2|}{|N^2|}$$

$$e_{max} = \frac{|Ne_1| + |Me_2|}{|N^2|}$$

$$e_{max} = \left|\frac{Ne_1}{N^2}\right| + \left|\frac{Me_2}{N^2}\right|$$

$$e_{max} = \left|\frac{e_1}{N}\right| + \left|\frac{Me_2}{N^2}\right|$$

$$e_{max} = \left|\frac{\text{M}}{N}\right|\left\{\left|\frac{e_1}{M}\right| + \left|\frac{e_2}{N}\right|\right\}$$

Alternatively

$$absolute\ error = \frac{1}{2}(max - min)$$

$$= \frac{1}{2}\left[\frac{(M+e_1)}{(N-e_2)} - \frac{(M-e_1)}{(N+e_2)}\right]$$

$$e_{M/N} = \frac{Me_2 + Ne_1}{N^2 - {e_2}^2}$$

Since e_1 and e_2 are very small, then

$${e_2}^2 \approx 0$$

$$\left|e_{M/N}\right| = \left|\frac{Me_2 + Ne_1}{N^2}\right|$$

$$\left|e_{M/N}\right| \leq \frac{|Me_2| + |Ne_1|}{|N^2|}$$

$$e_{max} = \frac{|Me_2| + |Ne_1|}{|N^2|}$$

$$e_{max} = \left| \frac{Ne_1}{N^2} \right| + \left| \frac{Me_2}{N^2} \right|$$

$$e_{max} = \left| \frac{e_1}{N} \right| + \left| \frac{Me_2}{N^2} \right|$$

$$e_{max} = \left|\frac{M}{N}\right| \left\{ \left| \frac{e_1}{M} \right| + \left| \frac{e_2}{N} \right| \right\}$$

10. (b)

$$t = \frac{d}{s} = \frac{6.43}{37.2} = 0.1728$$

$M = 6.43, \qquad e_1 = 0.005$

$N = 37.2, \qquad e_2 = 0.05$

$$e_{max} = \frac{|Me_2| + |Ne_1|}{|N^2|}$$

$$e_{max} = \frac{|6.43x0.05| + |37.2x0.005|}{|(37.2)^2|} = 0.000367$$

$t = 0.1728 \pm 0.000367$

Interval $[0.172, 0.173]$

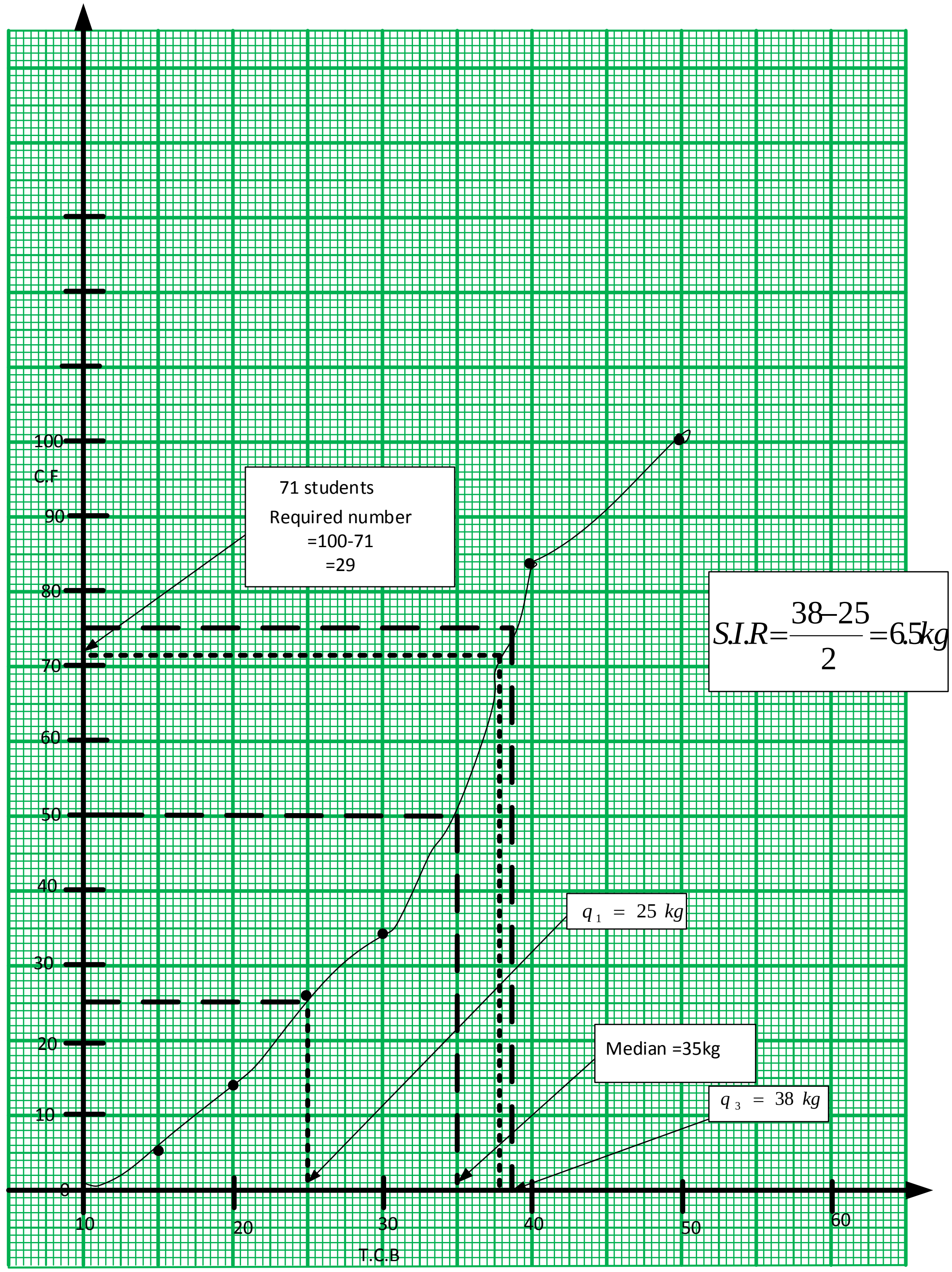
11.

weight	x	f	fx	fx^2
10 - < 15	12.5	5	62.5	781.25
15 - < 20	17.5	9	157.5	2756.25
20 - < 25	22.5	12	270	6075
25 - < 30	27.5	18	498	13612.5
30 - < 40	35	40	1400	49000
40 - < 50	45	16	720	32400
		Σf=100	Σfx=3108	Σfx^2=104625

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{3108}{100} = 31.08kg$$

$$S.D = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$S.D = \sqrt{\frac{104625}{100} - (31.08)^2} = 8.9601\text{kg}$$



C.F
100
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
T.C.B
71 students
Required number
=100-71
=29
$S.I.R = \frac{38-25}{2} = 6.5kg$
$q_1 = 25\ kg$
Median =35kg
$q_3 = 38\ kg$